Principles of Communications

EES 351

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4.3 Fourier Series

Office Hours:

Check Google Calendar on the course website.

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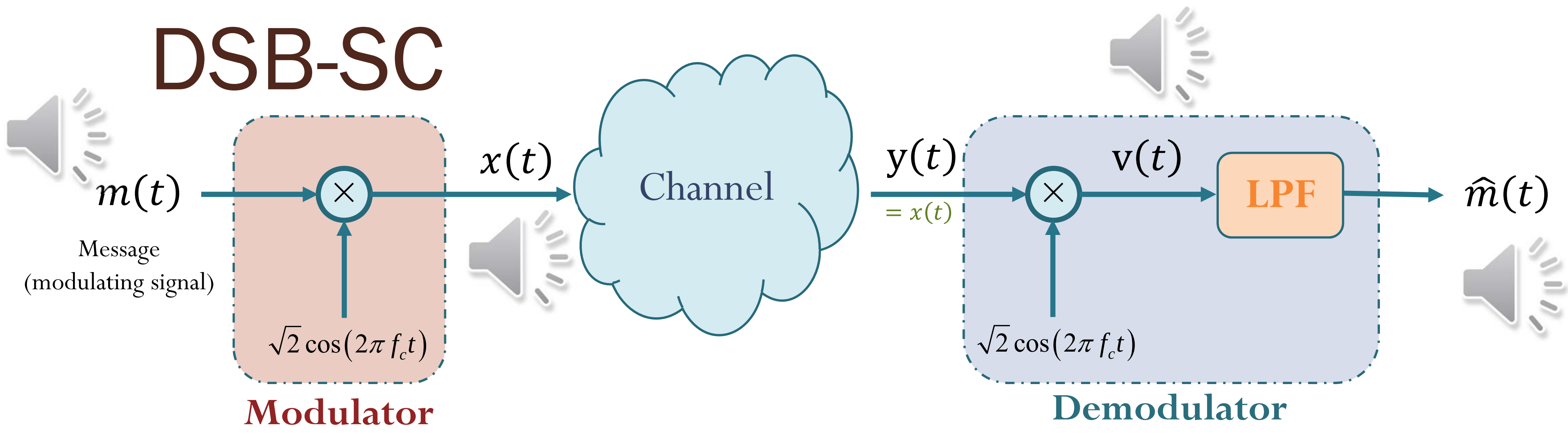
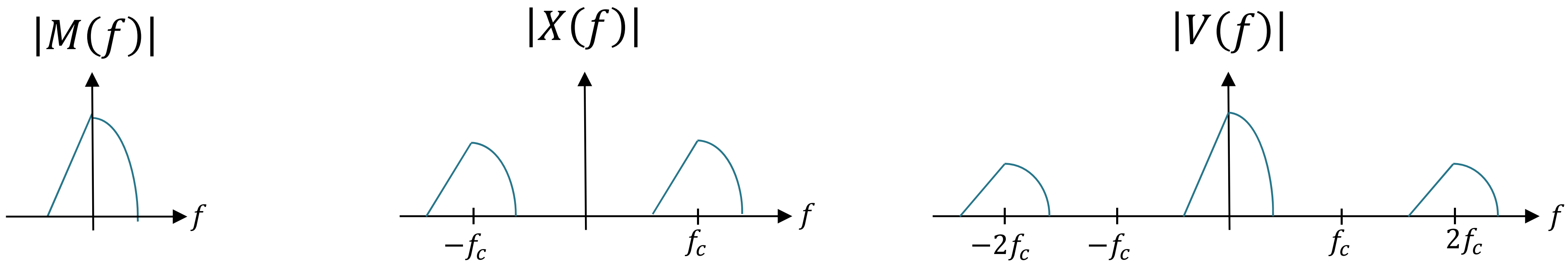
Section 4.3

- **Crucial Skill 4.3.1**: Find the Fourier series expansion of the periodic train of impulses and the periodic train of rectangular pulses.
- **Crucial Skill 4.3.2**: Understand the relationship between the "switching box" and "multiplication by rectangular pulse train".

DSB-SC





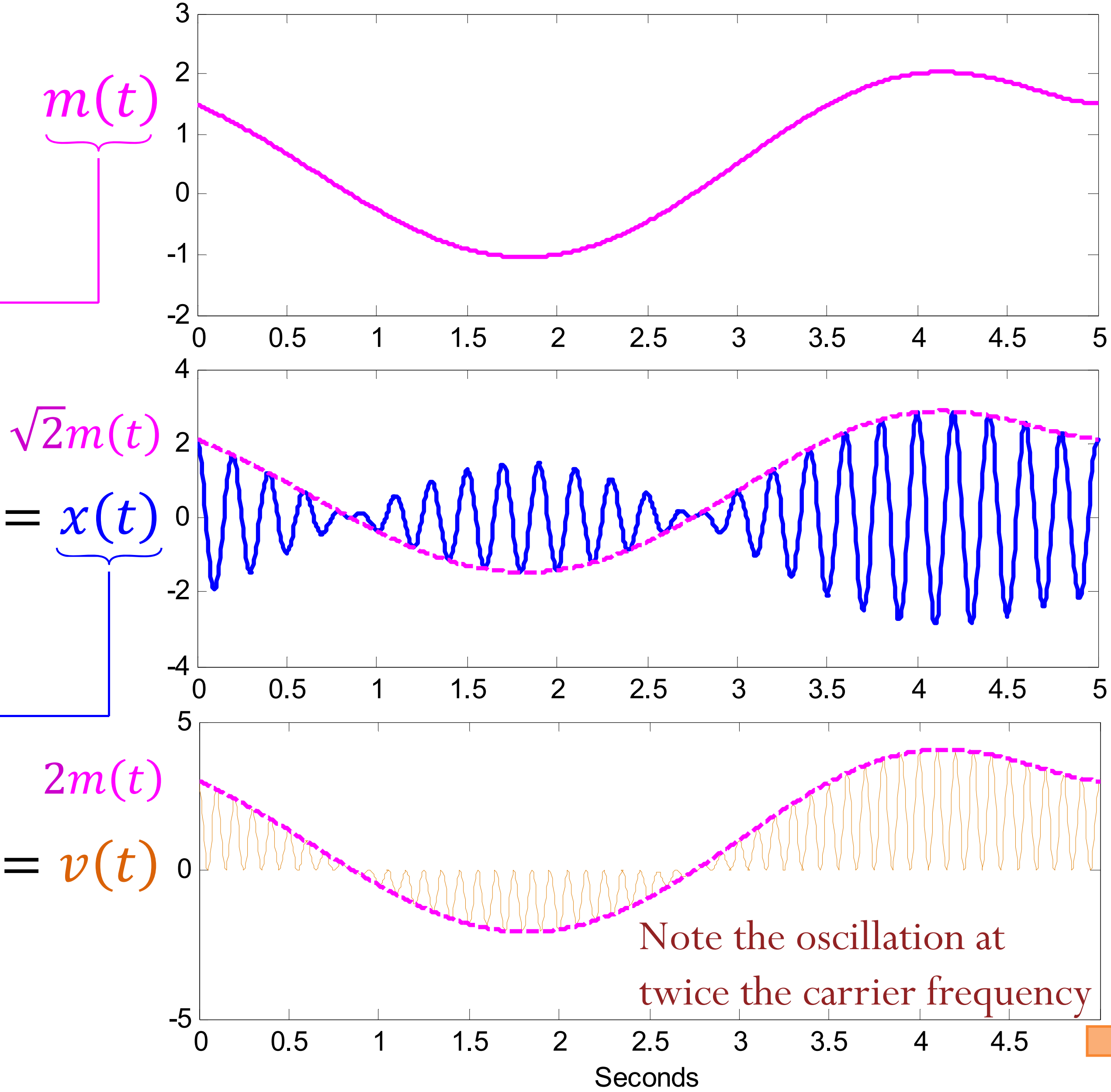
Key equation:

$$\text{LPF}\left\{ \underbrace{\underbrace{\left(m(t)\times\sqrt{2}\cos(2\pi f_c t)\right)}_{x(t)}\times\left(\sqrt{2}\cos(2\pi f_c t)\right)}_{v(t)} \right\} = m(t)$$

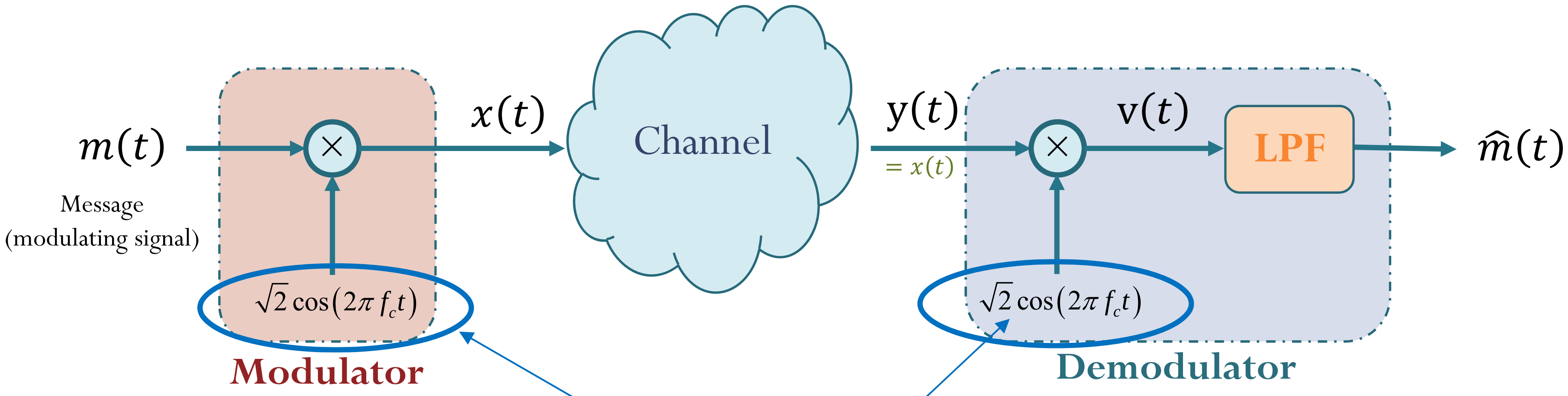
In the time domain…

$$m(t) \times \sqrt{2}\cos(2\pi f_c t) = x(t)$$

$$x(t) \times \sqrt{2}\cos(2\pi f_c t) = v(t)$$



A Problem for DSB-SC



It is not easy to synchronize these two oscillators. [4.10, 4.11]

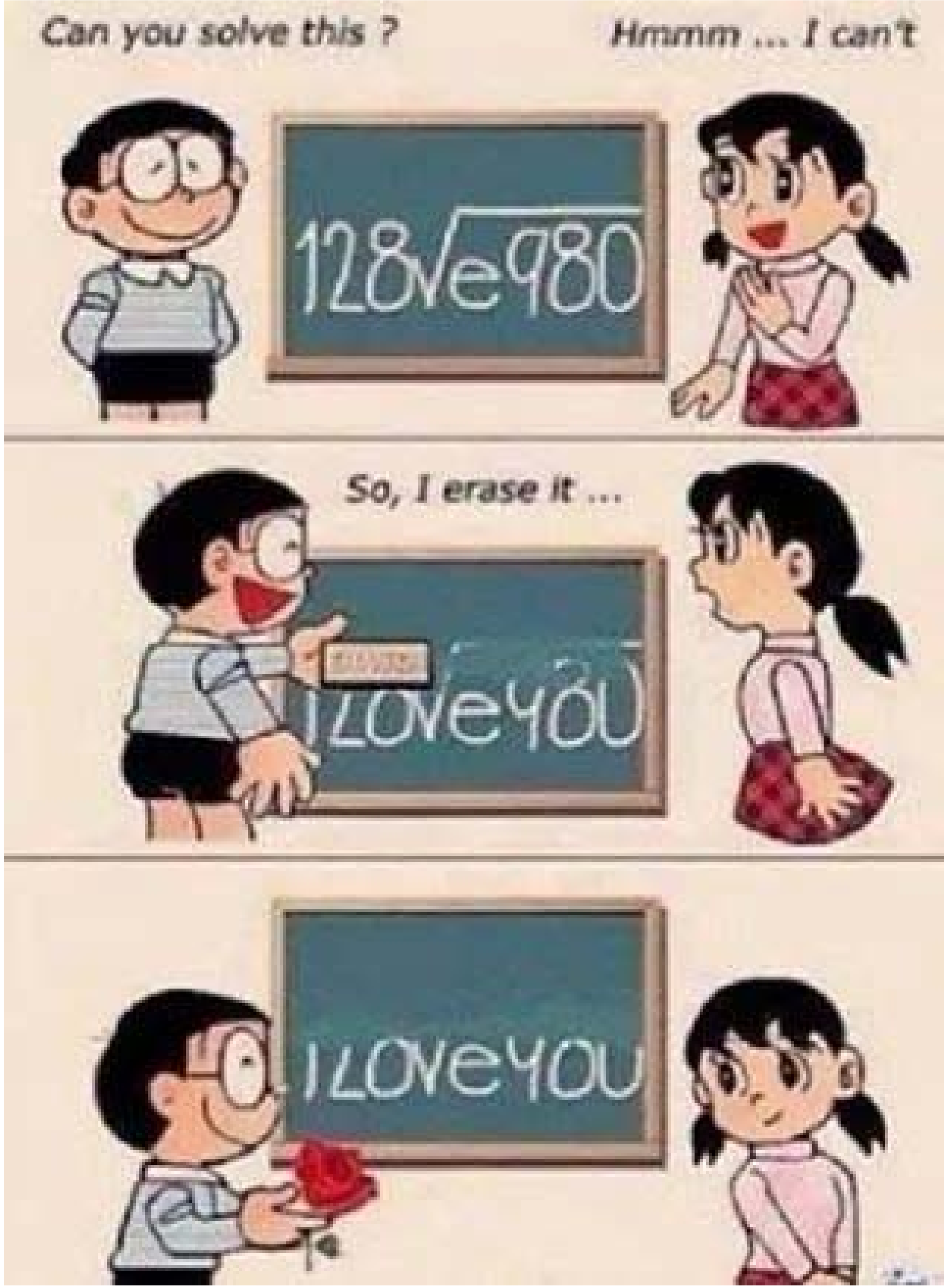
We need to find new solution.

Have you seen this before?

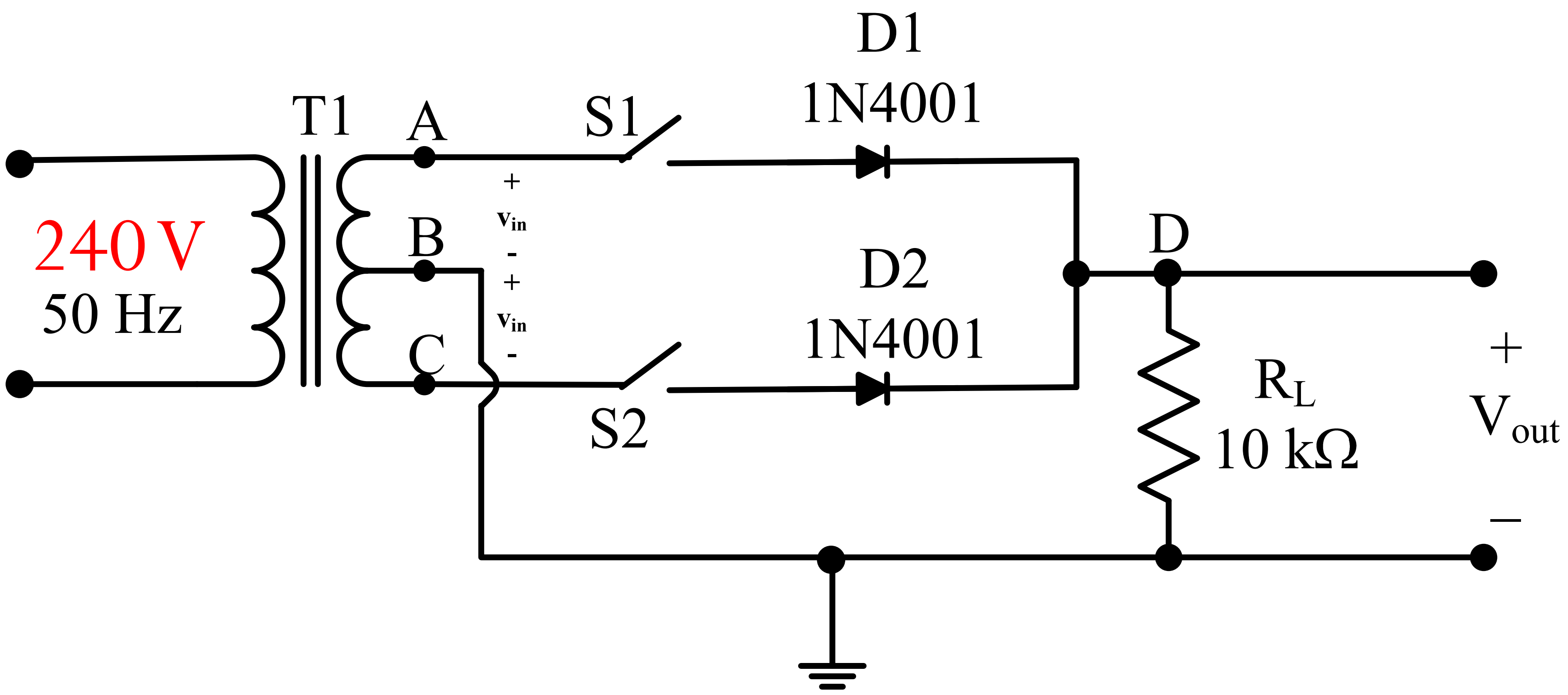
$$128\sqrt{e980}$$

Hint: Valentine's Day

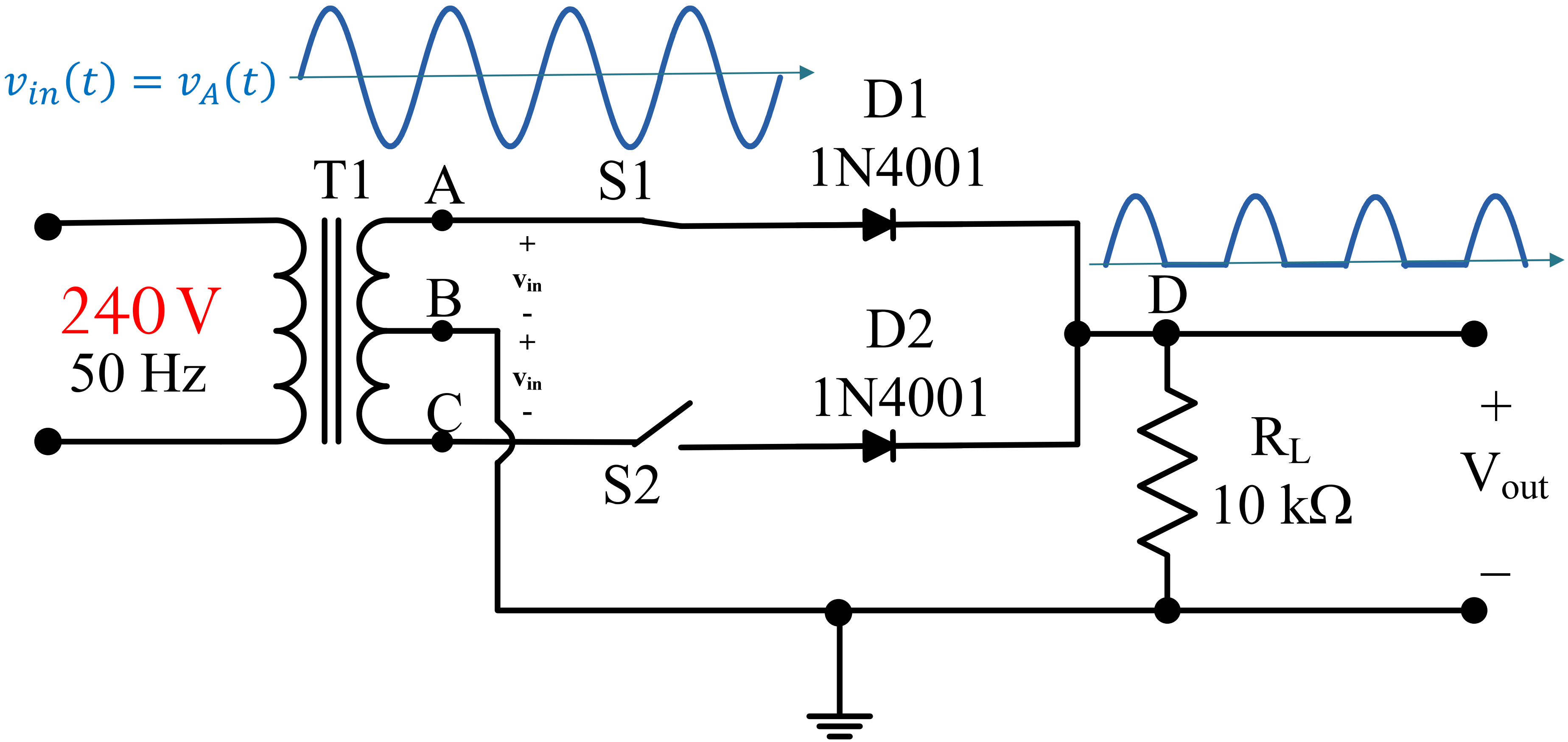

128√e980
SENORGIF.COM


Can you solve this ?
Hmmm ... I can't
128√e980
So, I erase it ...
I LOVE YOU
I LOVE YOU

Part A


T1
A
B
C
S1
S2
D1
1N4001
D2
1N4001
D
240 V
50 Hz
+
v_{in}
-
+
v_{in}
-
R_L
10 kΩ
+
V_{out}
–

Part A: Half-Wave Rectifier (HWR)



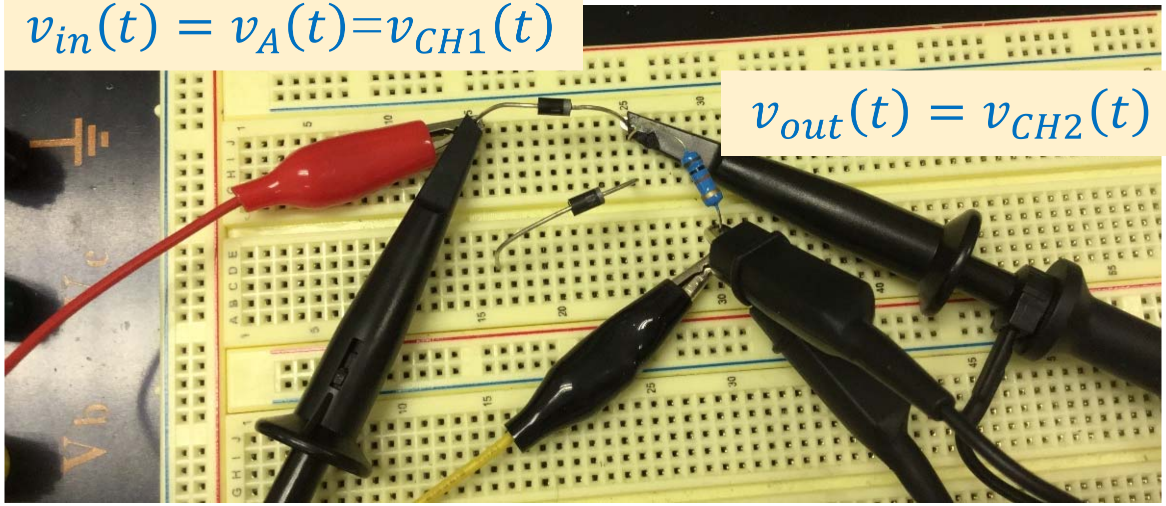
- **A rectifier** is an electrical device that converts alternating current (AC) to direct current (DC).

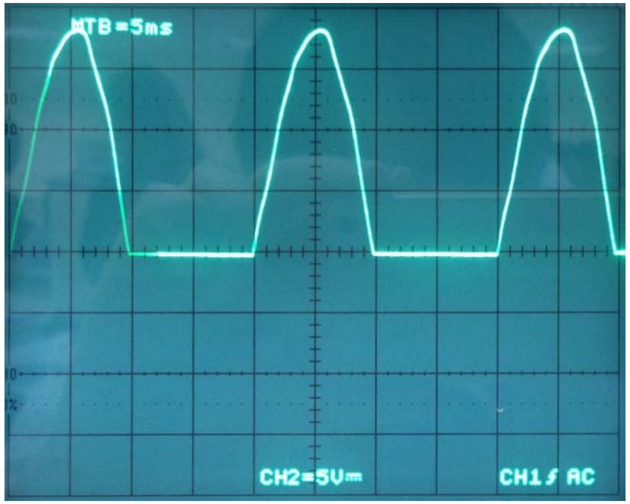
[Slides from basic EE lab]

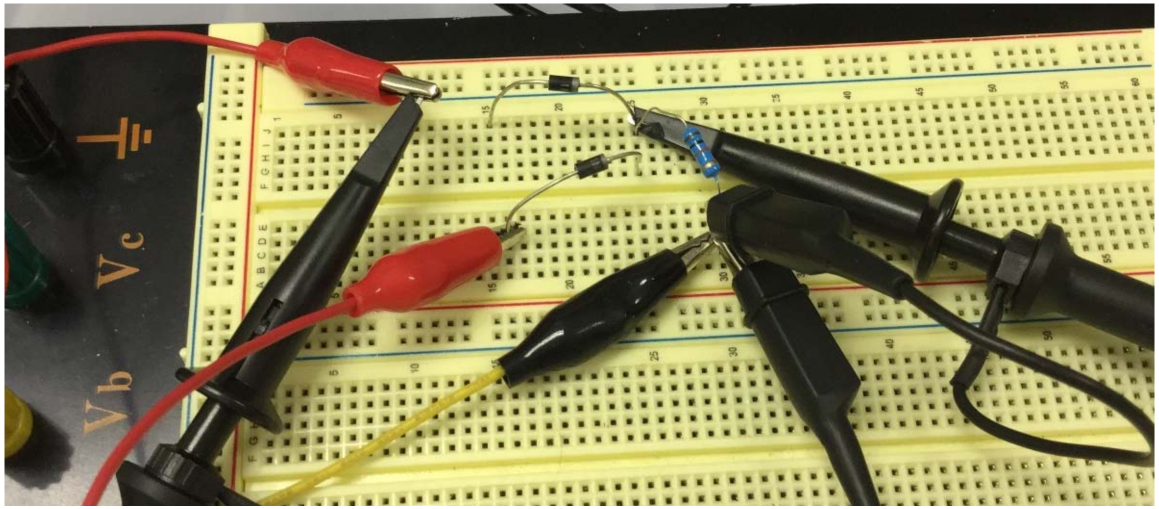
Part A: Half-Wave Rectifier (HWR)

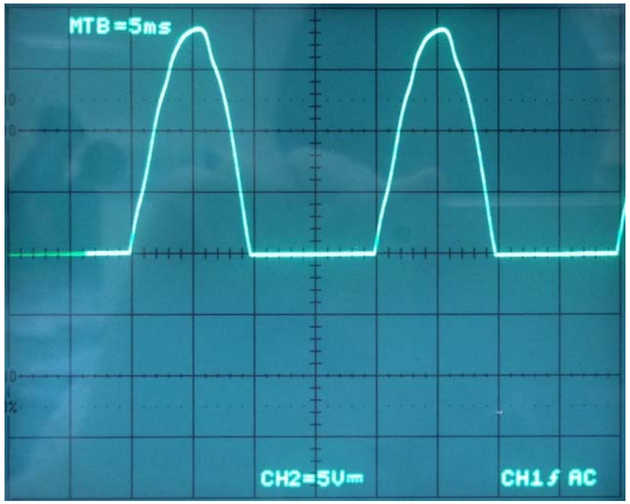
$v_{in}(t) = v_A(t) = v_{CH1}(t)$

$v_{out}(t) = v_{CH2}(t)$

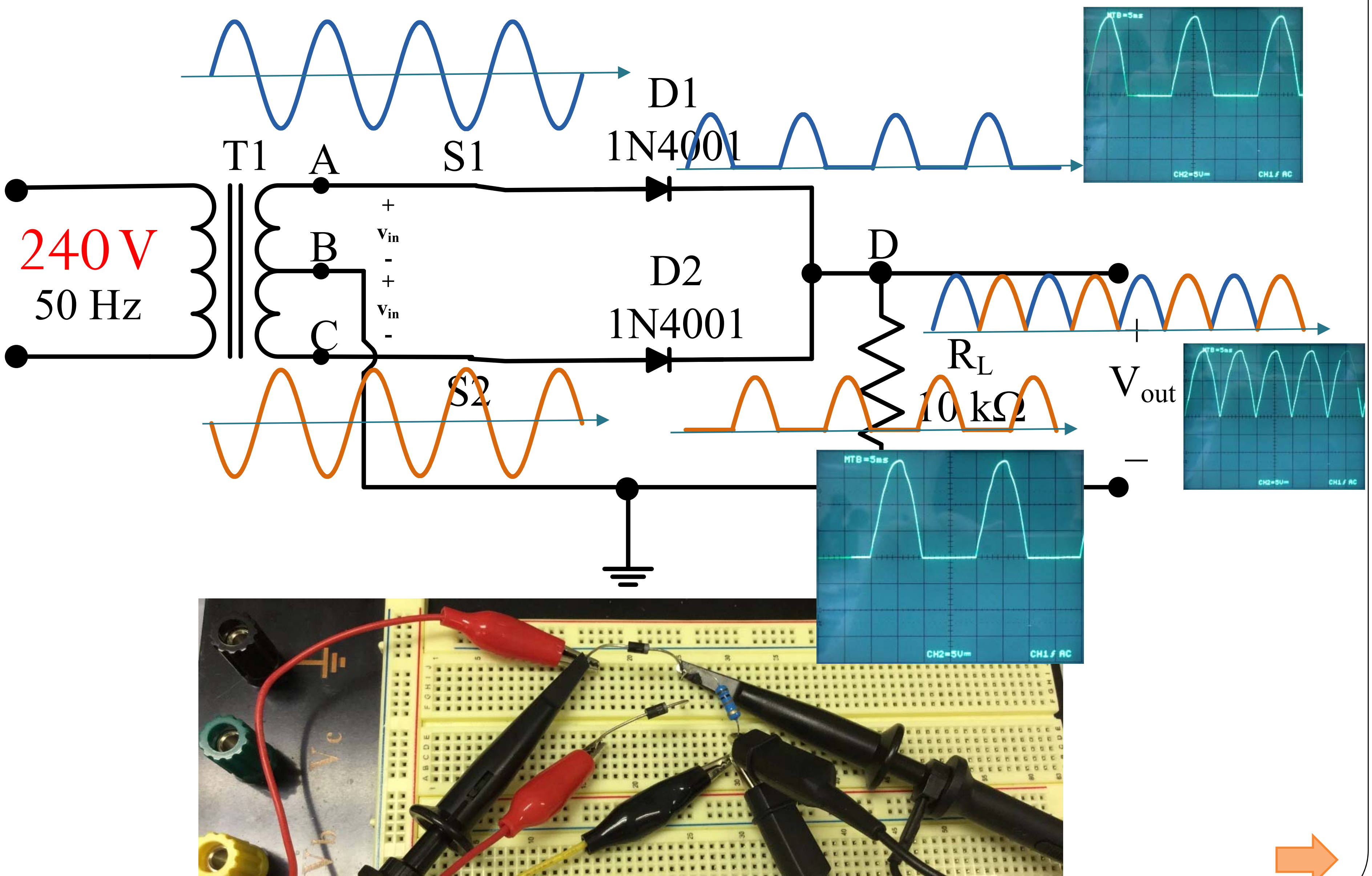






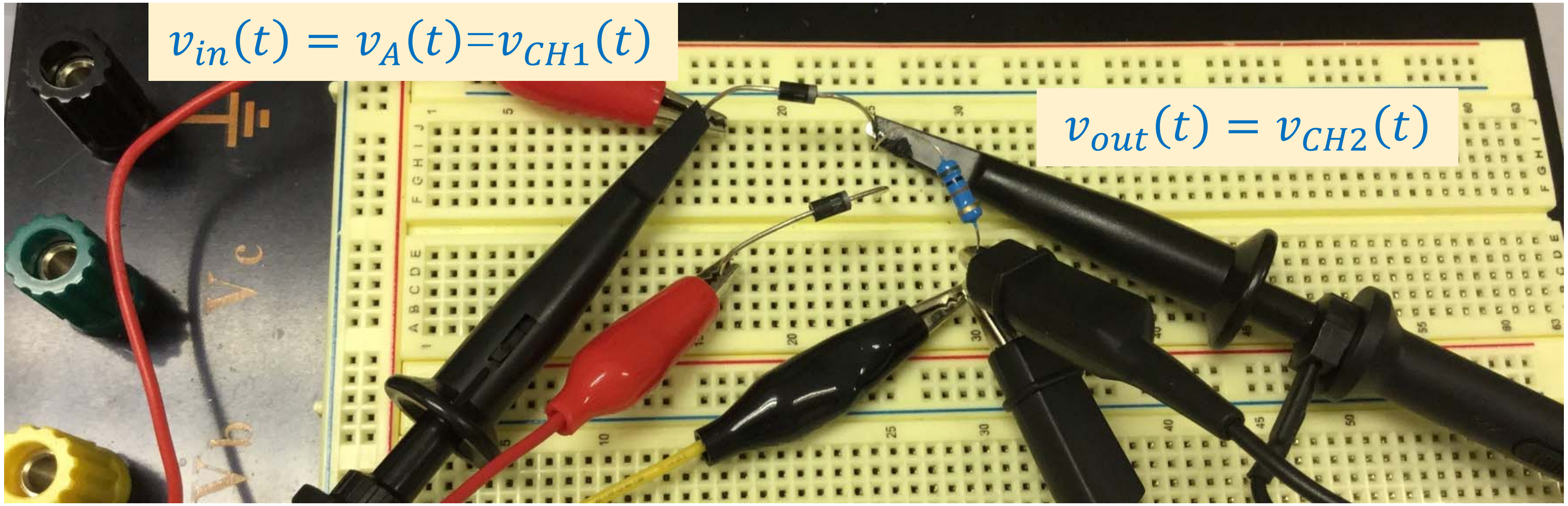


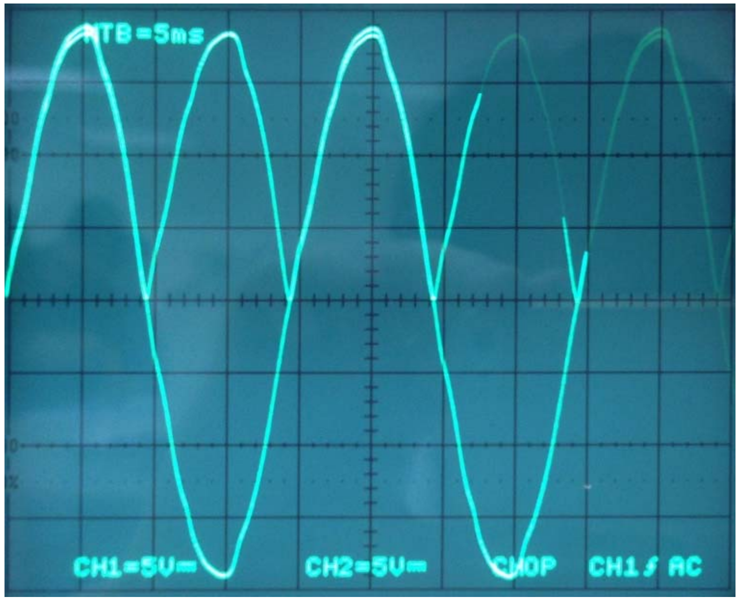
Part A: Full-Wave Rectifier (FWR)

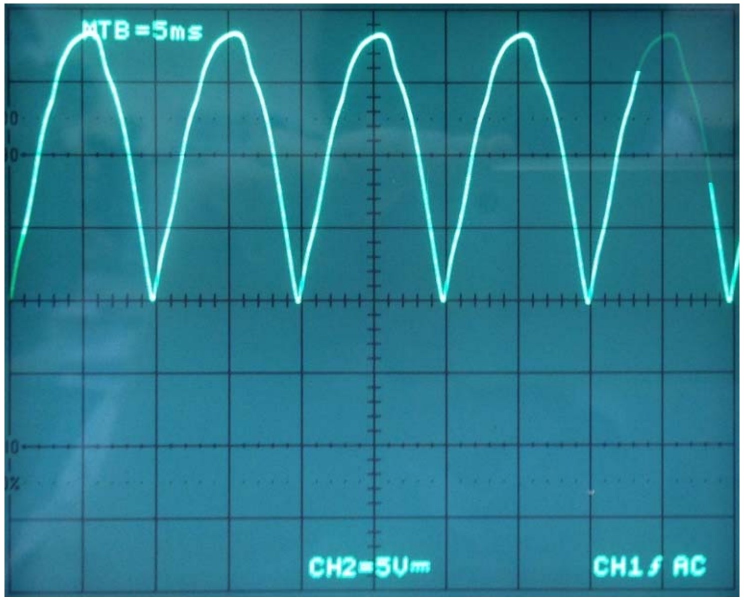


[Slides from basic EE lab]

Part A: Full-Wave Rectifier (FWR)

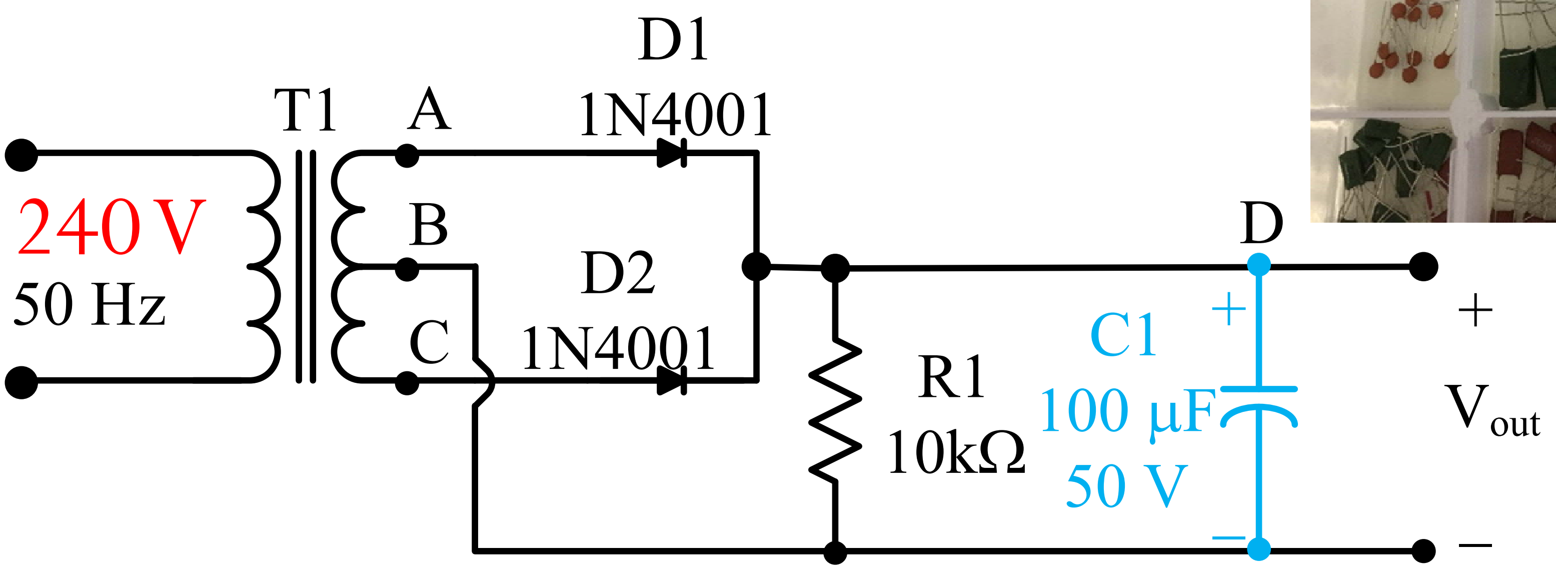


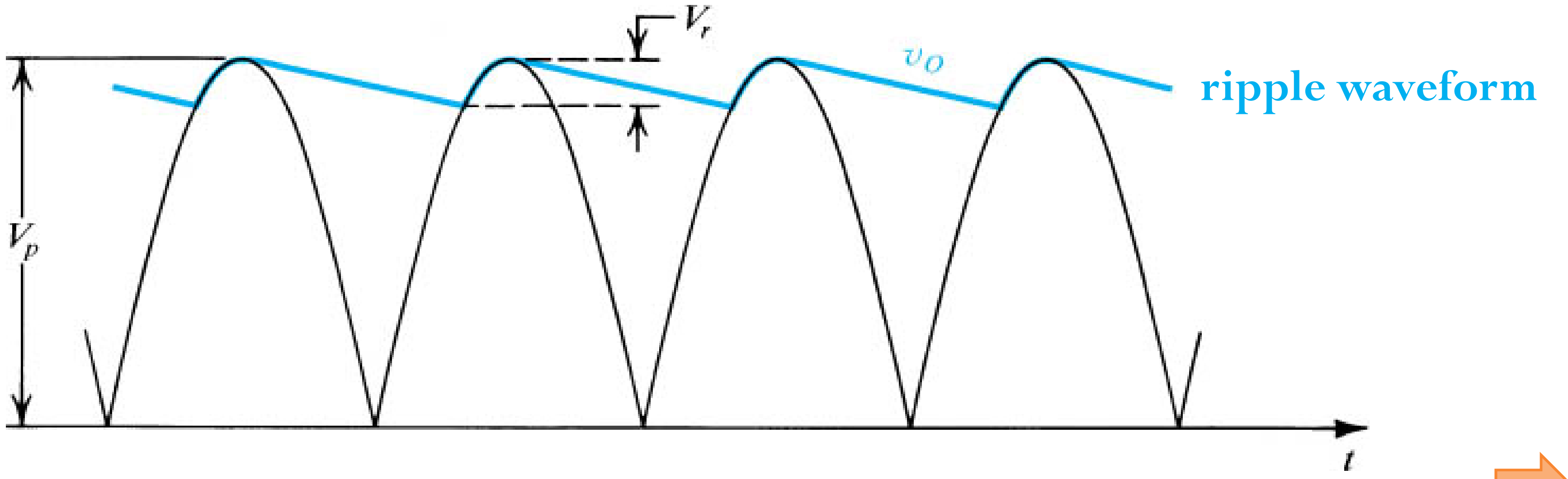




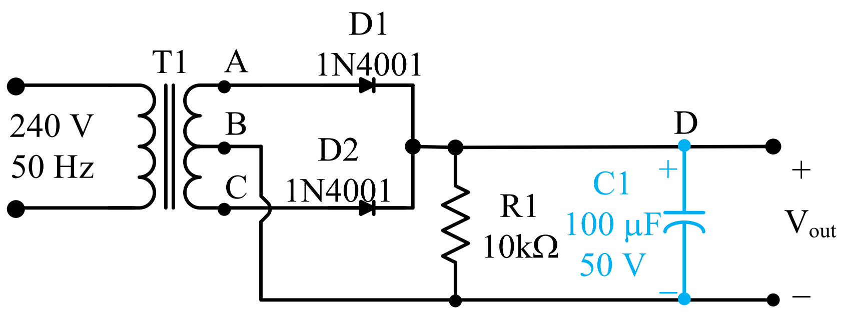
[Slides from basic EE lab]

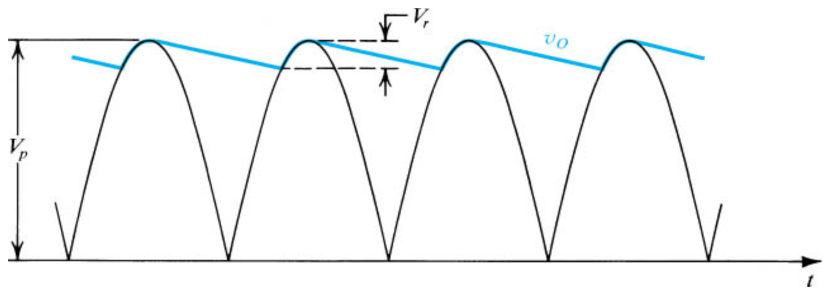
Part B: Filter Capacitor

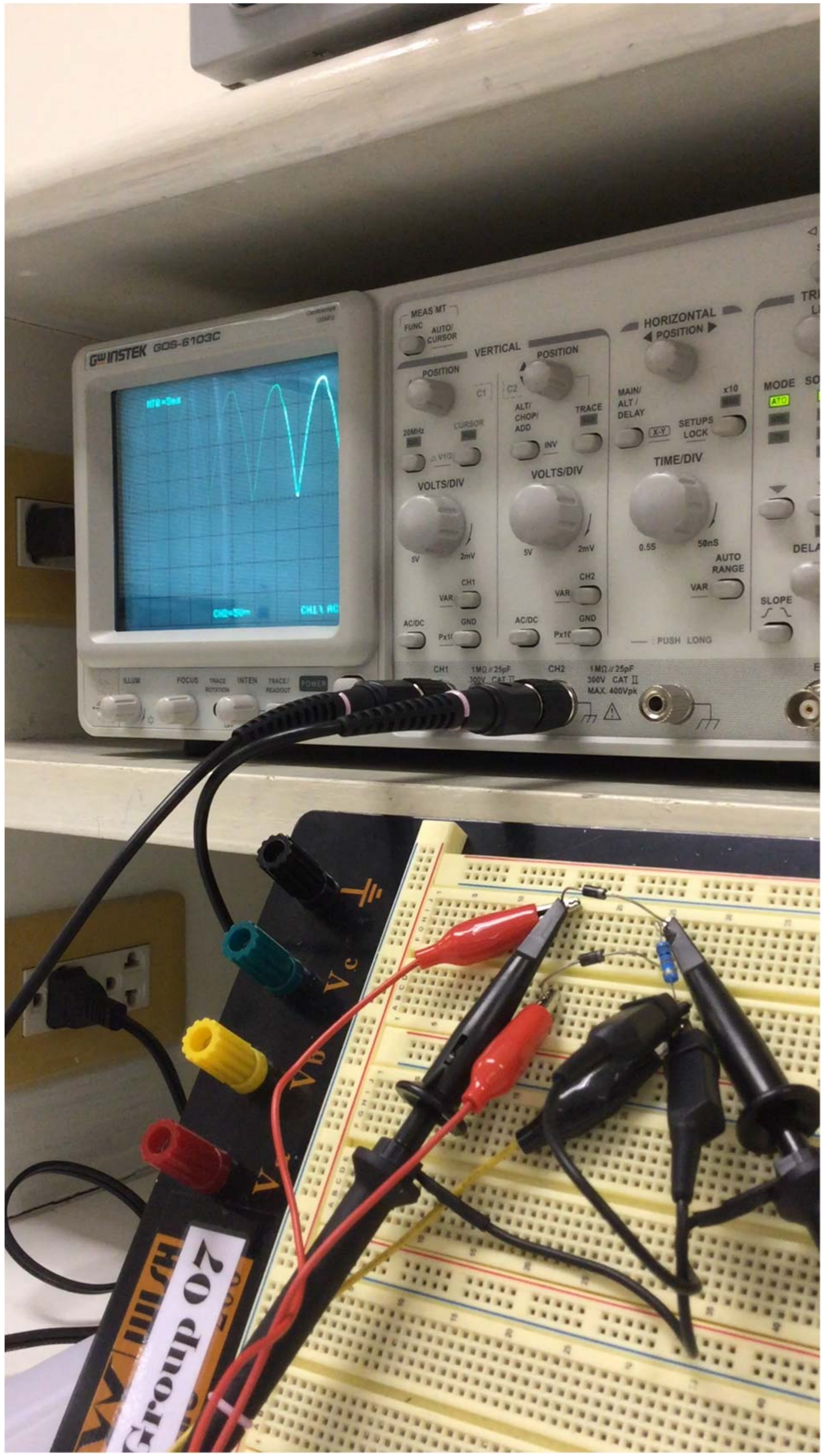




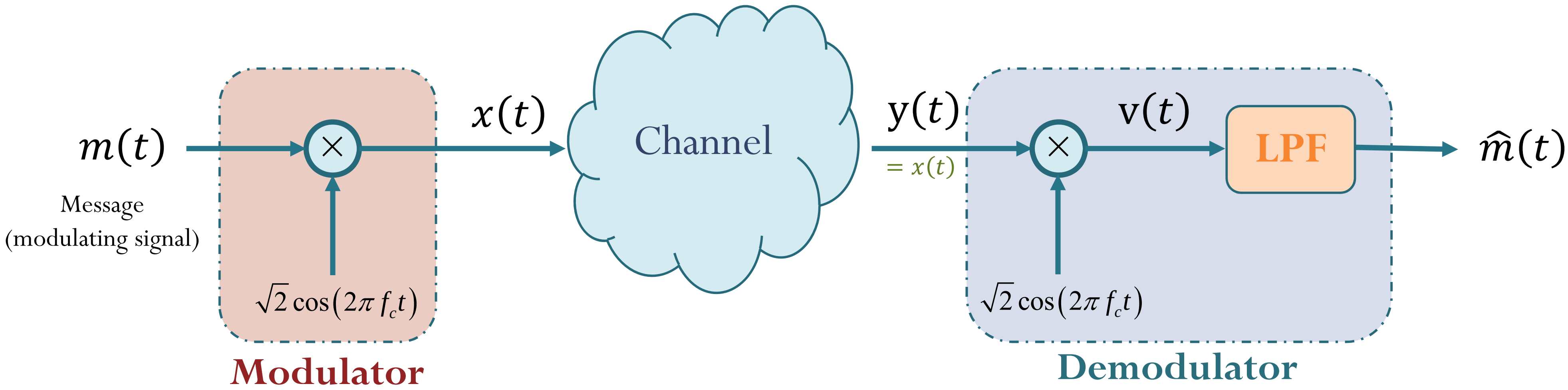
[Slides from basic EE lab]

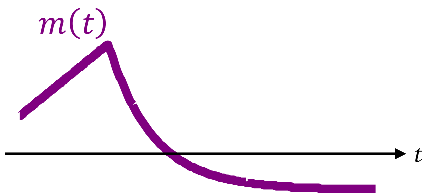


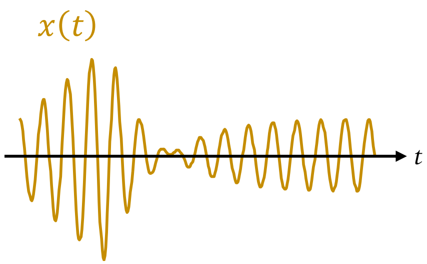




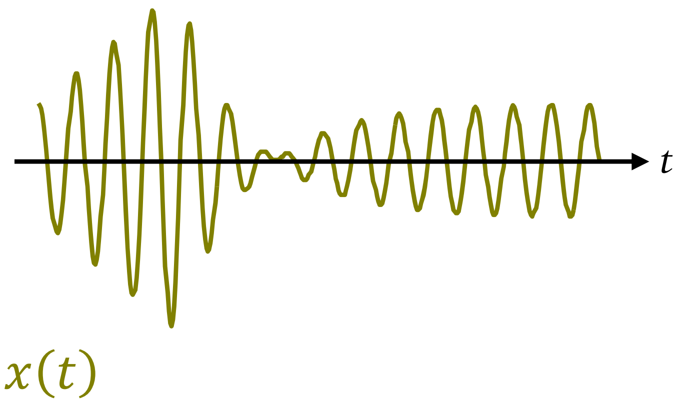
Back to DSB-SC

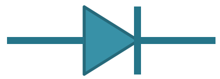




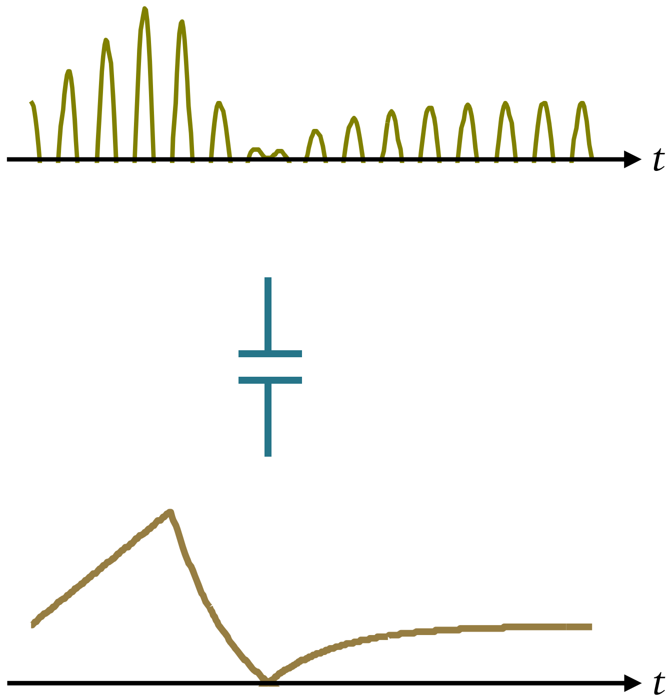


Envelope Detector

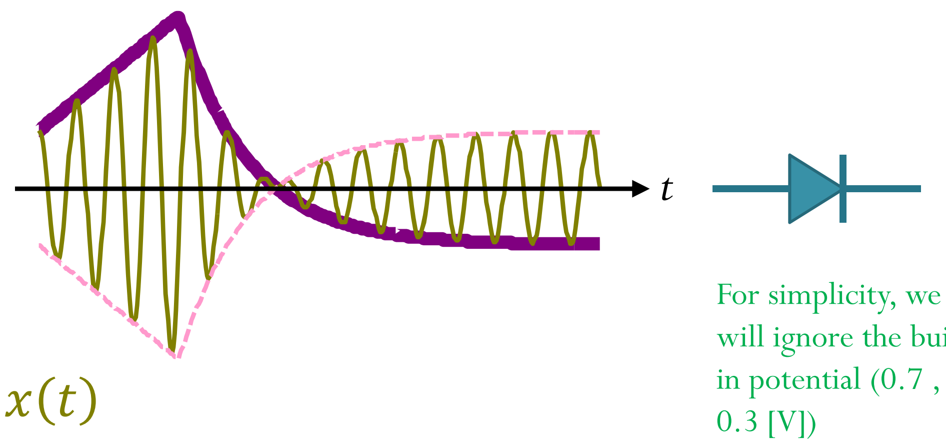




For simplicity, we will ignore the built-in potential (0.7 , 0.3 [V])



Envelope Detector



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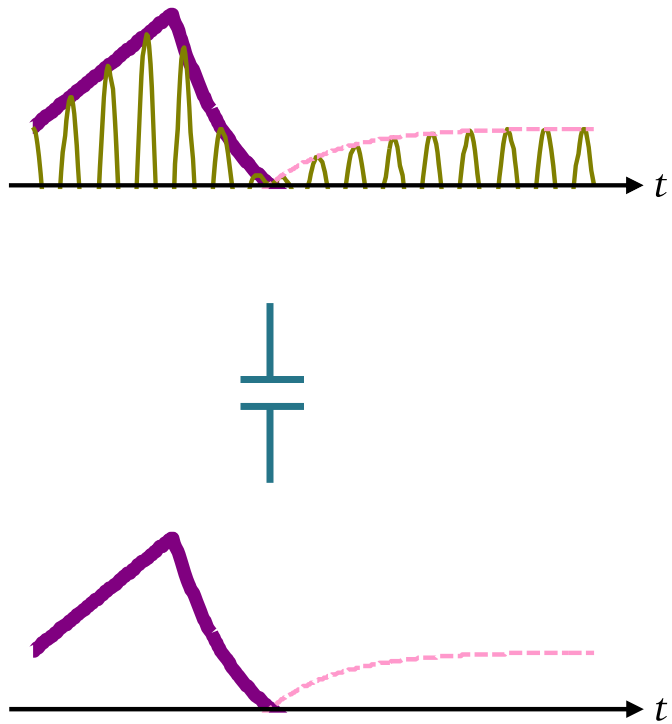
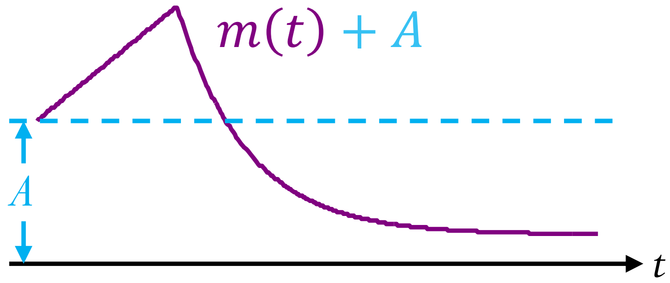
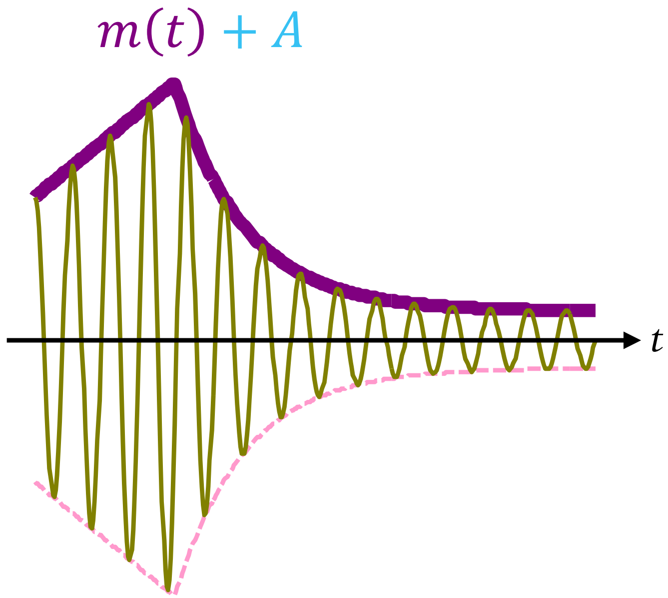
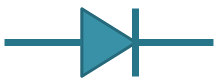
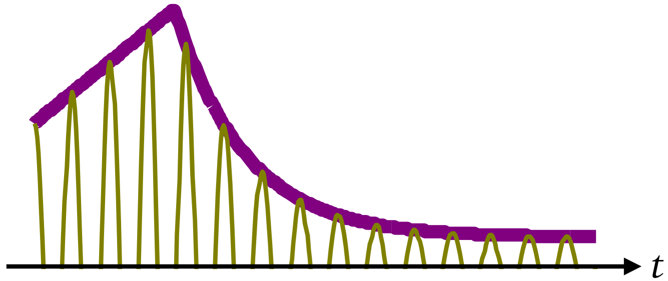


Envelope Detector









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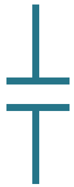
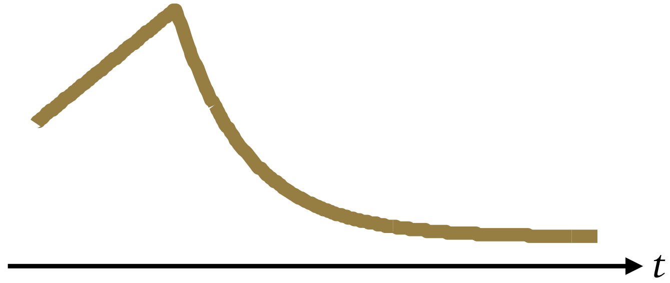
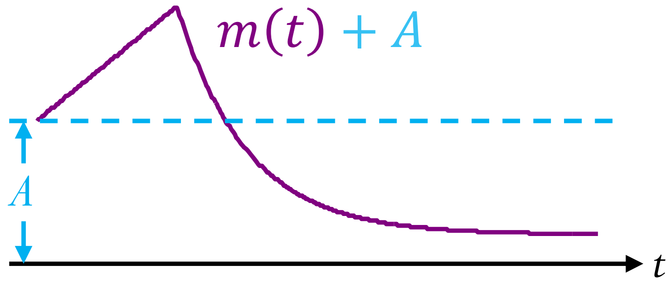
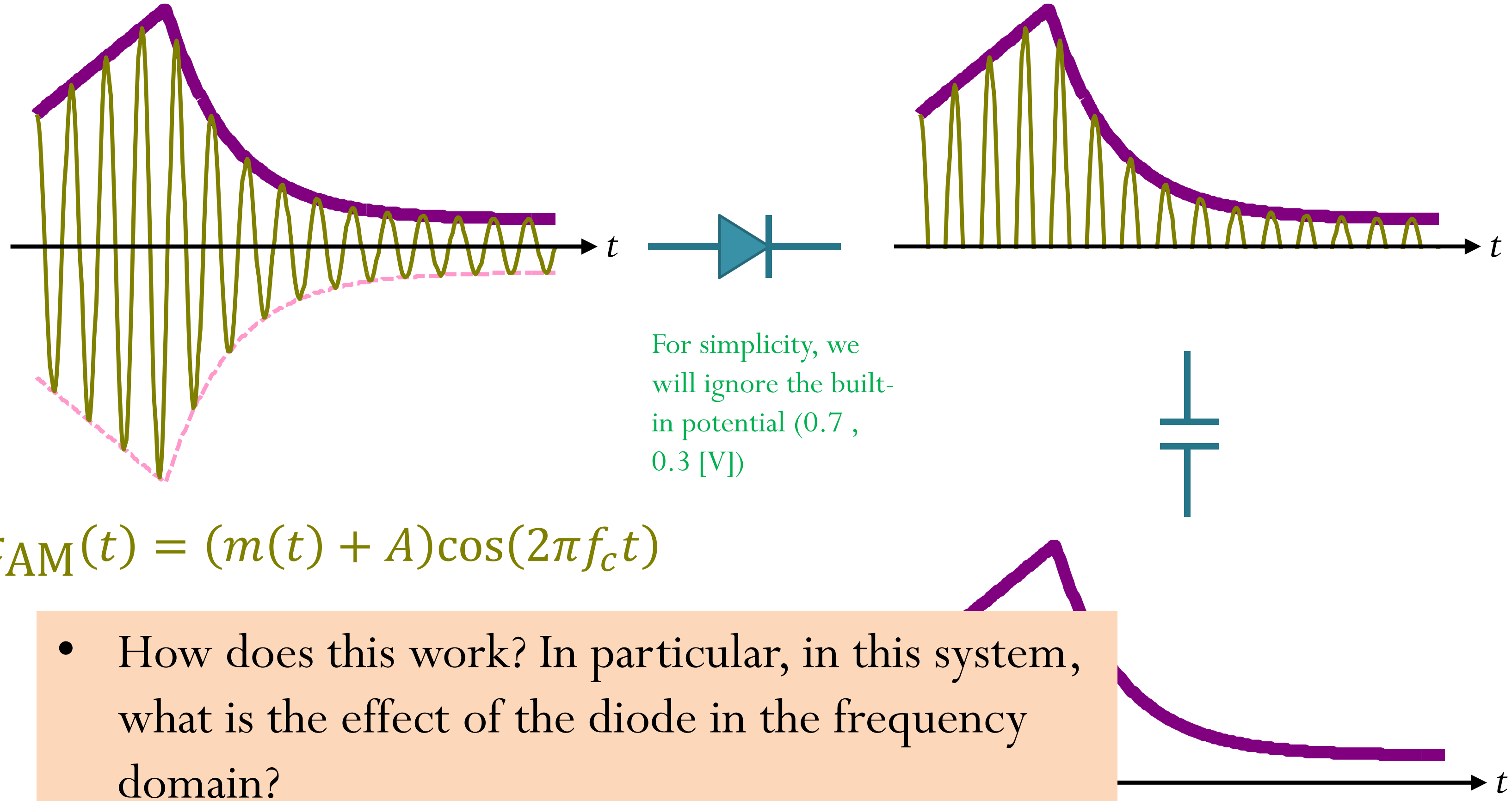




$$x_{\mathrm{AM}}(t) = (m(t) + A)\cos(2\pi f_c t)$$

Envelope Detector





$x_{\text{AM}}(t) = (m(t) + A)\cos(2\pi f_c t)$

- How does this work? In particular, in this system, what is the effect of the diode in the frequency domain?
- Can we replace the HWR (diode) by a FWR?
- Do we really need the diode? Can we remove it?

Fourier Series vs. Fourier Transform

Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi ft}df$$

"Linear combination" of complex-exponential functions

Fourier Series

$$r(t) \text{"="} \sum_{k=-\infty}^{\infty} c_k e^{j2\pi(kf_0)t}$$

[4.42]

Periodic with period $T_0 = \frac{1}{f_0}$

Fourier Series vs. Fourier Transform

Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df$$

Fourier Series

The main task in Fourier transform and Fourier series expansions is to find the values of these coefficients.

$$r(t) \text{"="} \sum_{k=-\infty}^{\infty} c_k e^{j2\pi(kf_0)t}$$

Periodic with period $T_0 = \frac{1}{f_0}$

Fourier Series vs. Fourier Transform

Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df \underset{}{\overset{\mathcal{F}}{\rightleftharpoons}} G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

Fourier Series

The main task in Fourier transform and Fourier series expansions is to find the values of these coefficients.

$$r(t) \text{"="} \sum_{k=-\infty}^{\infty} c_k e^{j2\pi (kf_0)t}$$

Periodic with period $T_0 = \frac{1}{f_0}$

[4.42] $$c_k = \frac{1}{T_0} \int_{T_0} r(t) e^{-j2\pi (kf_0)t} dt$$

Fourier Series vs. Fourier Transform

Fourier Transform

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi ft} df \xrightleftharpoons{\mathcal{F}} G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt$$

Fourier Series

[4.43]

$$r(t) \text{"="} \sum_{k=-\infty}^{\infty} c_k e^{j2\pi(kf_0)t} \xrightleftharpoons{\mathcal{F}} R(f) = \sum_{k=-\infty}^{\infty} c_k \delta(f - kf_0)$$

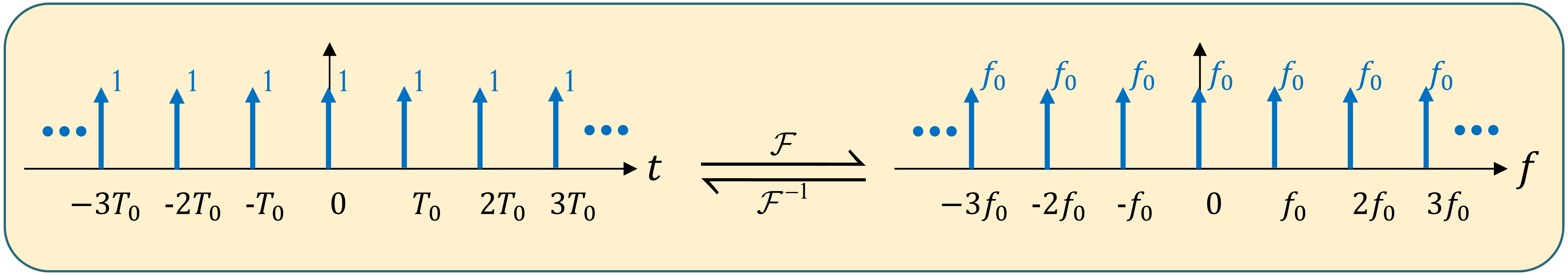
Periodic with period $T_0 = \frac{1}{f_0}$

Recall that a complex-exponential function corresponds to a delta function in the frequency domain at the freq. of the complex-expo. function.

$$c_k = \frac{1}{T_0} \int_{T_0} r(t) e^{-j2\pi(kf_0)t} dt$$

Periodic train of impulses

- Fourier series expansion of the periodic train of impulses:



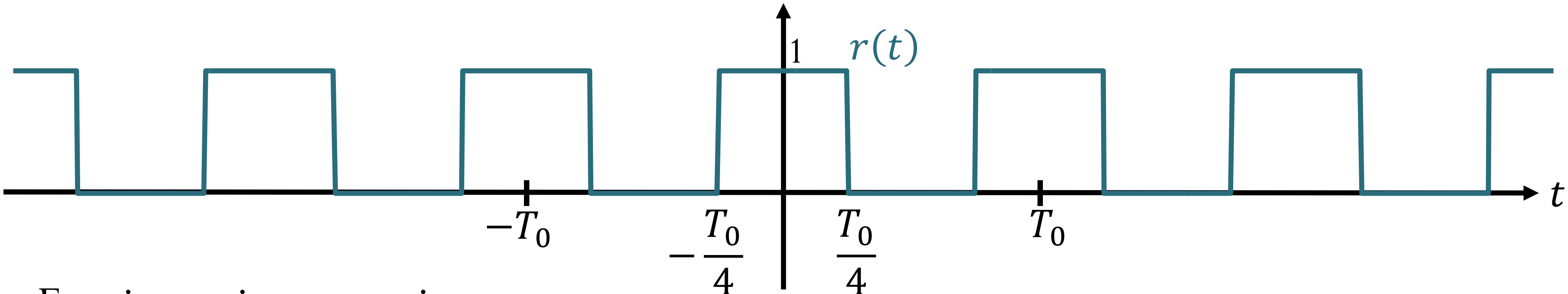
[4.47]

$$r(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} R(f) = \sum_{k=-\infty}^{\infty} f_0 \delta(f - kf_0)$$

$$= \sum_{k=-\infty}^{\infty} f_0 e^{j2\pi(kf_0)t}$$

[4.53] Square Wave

$$c_k = \frac{1}{2}\text{sinc}\left(k\frac{\pi}{2}\right) = \frac{1}{k\pi}\sin\left(k\frac{\pi}{2}\right)$$



Fourier series expansion:

$$r(t) = \frac{1}{2} + \frac{1}{\pi}e^{j(2\pi f_0 t)} - \frac{1}{3\pi}e^{j(2\pi(3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(5f_0)t)} + \cdots$$
$$+ \frac{1}{\pi}e^{j(2\pi(-f_0)t)} - \frac{1}{3\pi}e^{j(2\pi(-3f_0)t)} + \frac{1}{5\pi}e^{j(2\pi(-5f_0)t)} + \cdots$$

Trigonometric Fourier series expansion:

$$e^{jx} + e^{-jx} = 2\cos(x)$$

$$r(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_0 t) - \frac{2}{3\pi}\cos(2\pi(3f_0)t) + \frac{2}{5\pi}\cos(2\pi(5f_0)t) + \cdots$$

[4.48a] Switching Operation

Multiplying a signal $m(t)$ by the square-wave $r(t)$ is equivalent to switching $m(t)$ on (for half a period) and off periodically.